Properties of the Derivative

Note that $c \in \mathbb{R}$ is a constant and $f, g\colon \mathbb{R} \to \mathbb{R}$ are functions.

Linearity

$$\tfrac{d}{dx}[f(x) + g(x)] = \tfrac{d}{dx}[f(x)] + \tfrac{d}{dx}[g(x)] \qquad \tfrac{d}{dx}[c \cdot f(x)] = c \cdot \tfrac{d}{dx}[f(x)]$$

Product Rule

$$\frac{d}{dx}[f(x) \cdot g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)]$$

Chain Rule

$$\frac{d}{dx}[f(g(x))] = \frac{d}{dg}[f(g(x))] \cdot \frac{d}{dx}[g(x)]$$

Common Derivatives

Note that $a, c \in \mathbb{R}$ and $n \in \mathbb{Z}$ are constants.

Polynomials

$$\tfrac{d}{dx}[c] = 0 \qquad \tfrac{d}{dx}\left[x^n\right] = nx^{n-1}$$

Exponentials and Logarithms

$$\tfrac{d}{dx}\left[a^x\right] = \ln(a) \cdot a^x \qquad \tfrac{d}{dx}\left[\log_a(x)\right] = \tfrac{1}{\ln(a)} \cdot \tfrac{1}{x}, \text{ (for } x > 0)$$

Trigonometric Functions

$$\tfrac{d}{dx}[\sin(x)] = \cos(x) \qquad \tfrac{d}{dx}\left[\cos(x)\right] = -\sin(x)$$

Inverse Trigonometric Functions

$$\tfrac{d}{dx}\left[\sin^{-1}(x)\right] = \tfrac{1}{\sqrt{1-x^2}} \qquad \tfrac{d}{dx}\left[\cos^{-1}(x)\right] = -\tfrac{1}{\sqrt{1-x^2}}$$

$$\tfrac{d}{dx}\left[\tan^{-1}(x)\right] = \tfrac{1}{1+x^2} \qquad \tfrac{d}{dx}\left[\cot^{-1}(x)\right] = -\tfrac{1}{1+x^2}$$

$$\tfrac{d}{dx}\left[\sec^{-1}(x)\right] = \tfrac{1}{|x|\sqrt{x^2-1}} \qquad \tfrac{d}{dx}\left[\csc^{-1}(x)\right] = -\tfrac{1}{|x|\sqrt{x^2-1}}$$

Hyperbolic Functions

$$\tfrac{d}{dx}[\sinh(x)] = \cosh(x) \qquad \tfrac{d}{dx}\left[\cosh(x)\right] = \sinh(x)$$